

## NEW PROCEDURES FOR 2-D AND 3-D MICROWAVE CIRCUIT ANALYSIS WITH THE TLM METHOD

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### ABSTRACT

This paper contains four original contributions to numerical field modeling with the TLM method:

1. The formulation of a 3-D "Johns Matrix" (or Numerical Green's Function) for wideband non-TEM absorbing boundary conditions using the 3-D Condensed TLM node.
2. Use of tapered "Johns Matrix" (or Numerical Green's Function) for the improvement of the return loss of frequency dispersive absorbing boundaries.
3. A recursive algorithm for wideband non-TEM absorbing boundary modeling.
4. A pseudo-parallel iteration scheme for the simultaneous processing of TLM substructures.

These procedures are essential for efficient time domain modeling of 3-D waveguide discontinuities of arbitrary geometries. Their application saves considerable computer run time and memory when compared with conventional TLM analysis.

### 1. INTRODUCTION

The Transmission Line Matrix (TLM) method has been used since 1971 for the analysis of complex microwave structures. It is a time domain numerical method in which both space and time are discretized [1]-[2]. Either shunt or series connection of transmission lines can be used for 2-D analysis. Recently, we have developed several new concepts and procedures to speed up 2-D TLM modeling [3]. To analyse 3-D problems, the expanded node and asymmetrical condensed node [2] have been in use for some time. The former is topologically complicated because of the spatial separation of the field components. The latter defines all six field quantities at single points in space but has the disadvantage of being asymmetrical. Recently, a symmetrical 3-D condensed node (shown in Fig. 1) has been developed by P. B. Johns [4]. This node avoids the above problems and is more accurate than the other mesh schemes. The work presented in this paper is therefore based on this condensed node.

Sofar, there have been no reports on the computation of microwave scattering parameters with this node. To extract the scattering parameters over a wide range of frequencies from a single TLM simulation, we must model wideband absorbing boundaries in the time domain. Without them, the impulse excitation capability which is one of the main assets of the TLM method, cannot be exploited. Furthermore, the wideband absorbing boundaries must be of high quality since the Fourier transform of time domain results is very sensitive to imperfect boundary treatment. We have implemented wideband absorbing boundary conditions using the time domain Diakoptics approach [5].

### 2. MODELING OF ABSORBING BOUNDARY CONDITIONS

Our objective is to compute the scattering parameters of a 3-D discontinuity or a set of discontinuities in a waveguiding structure. To this end, we must compute the incident, reflected and transmitted fields at the reference planes indicated in Fig. 2. We assume that only the dominant mode of the embedding structure exists at these reference planes over a given frequency band. The space between the two reference planes is modeled by a 3-D TLM condensed node mesh. The absorbing boundary conditions must be implemented at the reference planes. These must simulate the extension of the waveguide to infinity away from these planes. To achieve this we proceed in two steps:

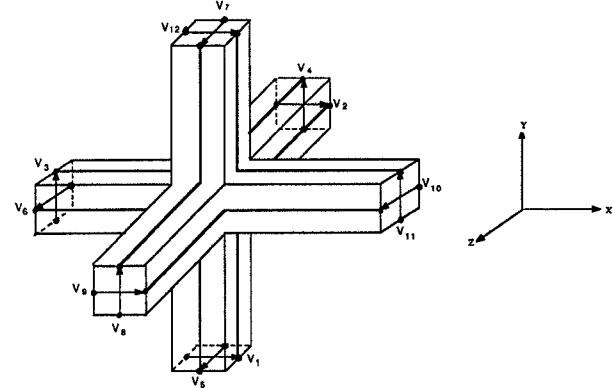


Fig. 1 : Three-Dimensional Condensed TLM node developed by Johns [4]

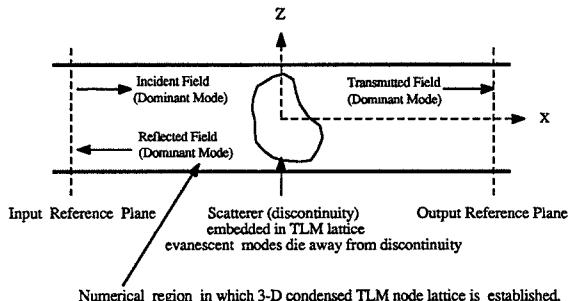


Fig. 2 : Discontinuity in a Waveguide Section

- a) We compute the impulse response or numerical Green's function at the input of a very long waveguide section, and stop the computations before the reflections from the far end return to the reference plane. For example, for a computation covering 2000 iterations, we need to discretize a waveguide section which is 500  $\Delta l$  long (because the velocity of waves on the TLM mesh is half the velocity of pulses on the individual mesh transmission lines). This numerical Green's function is kept in store.
- b) We then discretize the structure (shown in Fig. 2) between the reference planes, excite it at one end, and convolve the impulses emerging from these planes with the numerical Green's functions computed above. These two procedures are explained in detail below.

#### 2.1 Computation of Impulse Response (Johns Matrix) of a Long Waveguide

A long section of waveguide is discretized with 3-D TLM condensed nodes. Note that for  $TE_{10}$  mode propagation, the pulse values on branches 6, 10, 2 and 9 of condensed nodes are always zero (be-

cause  $E_x$  and  $E_z$  are zero). Hence we have non zero pulse values only on the remaining 8 branches. Furthermore, since there is no variation along  $y$ , we need to take only one node in the  $y$ -direction. We inject impulses (whose magnitudes vary as  $\sin(\pi z/a)$  along the  $z$ -direction) into branch 3 of all the nodes along the input reference plane. This will cause impulses separated by two times the iteration time interval to flow in streams out of the branch 3 of all the nodes along the input reference plane of this structure. These impulse functions result from the scattering at the nodes and boundaries of the structure, and can be interpreted as a Green's function in numerical form. We store the impulses on branch 3 of condensed node in the center of the waveguide cross-section. We call this numerical Green's function a "Johns Matrix" in honour of the late P. B. Johns, pioneer of TLM and time domain diakoptics. Note that the Johns Matrix is computed only once and stored.

## 2.2 Convolution with Impulse Response Johns Matrix

We excite the circuit (shown in Fig. 2) at branch 3 of all the nodes along the input reference plane with impulses whose magnitudes are spatially distributed according to the dominant mode. These impulses are scattered at nodes and boundaries and reach, after some time, the input and output reference planes. We store the impulses arriving on branch 3 of the center node on the input reference plane and branch 11 of the center node on the output reference plane. Then the reflected impulse voltages on these branches are computed by convolving the incident impulses with the Johns Matrix computed previously:

$$V_{11}^r(k) = \sum_{k'=0}^k J(k') \times V_{11}^i(k - k') \quad (1)$$

$$V_3^r(k) = \sum_{k'=0}^k J(k') \times V_3^i(k - k') \quad (2)$$

where  $J$  is the Johns Matrix.

Since we know the transverse field distribution of the propagating mode (e.g. variation for the  $TE_{10}$  mode in rectangular waveguides is  $\sin(\pi z/a)$ ), the reflected impulses at the other nodes in the reference planes can be calculated from those at the center.

Following the above approach, we have computed the reflections of two absorbing boundaries separated by a WR28 waveguide section (about  $60 \Delta t$  long). The magnitude of reflections obtained as  $\frac{V_{SWR-1}}{V_{SWR+1}}$  is shown in Fig. 3. It varies from 6 to 2 percent over the operating band of the waveguide. The S-parameters of a symmetrical inductive iris (of gap width equal to 3.46 mm) in a WR28 waveguide computed with these absorbing boundary conditions are compared with those computed using Marcuvitz's [6] in Fig. 4. Note the ripple in the TLM results, especially in the phase characteristics of the S-parameters. Hence we conclude that the quality of the absorbing boundaries described by the Johns matrix of a long section of a uniform guide is not good enough for S-parameter extraction. In the following, we show how these boundary conditions can be improved by "tapering" the Johns Matrix response in the time dimension.

## 3. TAPERED IMPULSE RESPONSE OR JOHNS MATRIX

In the case of 2-D TLM absorbing boundary algorithms, we have noticed that a waveguide termination with gradually increasing losses (like in practical waveguide terminations) gives better performance than a long uniform guide. This may be due to the absorption of the stray reflections due to the finite space and time discretization steps  $\Delta t$  and  $\Delta t$ . But the present 3-D condensed node cannot account for losses. However, we know that for homogeneous lossy material, the output impulse response value  ${}_k A'_i$  for electric and magnetic fields at any node and at any instant  $k \Delta t$  is related to the value  ${}_k A_i$  in the lossless case as follows [2]:

$${}_k A'_i = {}_k A_i e^{(-k\alpha\Delta t)} \quad (3)$$

where  $\alpha$  is the attenuation constant of the mesh lines. Thus by just recalculating the impulse response using different attenuation constants  $\alpha$ , different loss conditions can be covered with a single simulation. Following this argument, the Johns Matrix ( $J'(k)$ ) for a long uniform guide with constant loss can be related to the Johns Matrix ( $J(k)$ ) for a long lossless uniform guide as follows:

$$J'(k) = J(k) e^{(-k\alpha\Delta t)} \quad (4)$$

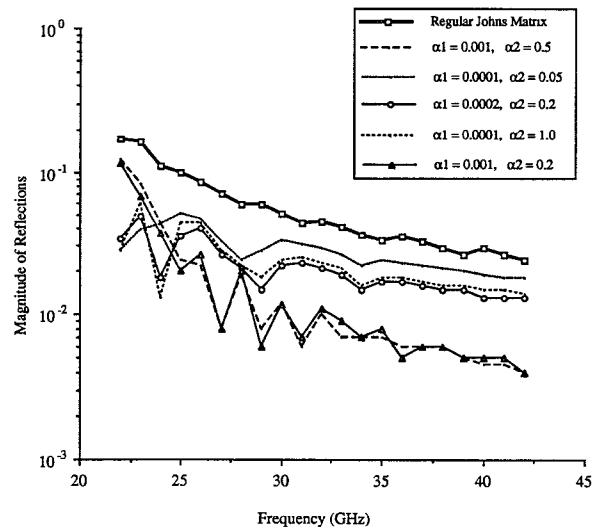


Fig. 3 : Reflection Characteristics of Absorbing Boundaries (WR 28) represented by Regular and Tapered Johns Matrices.

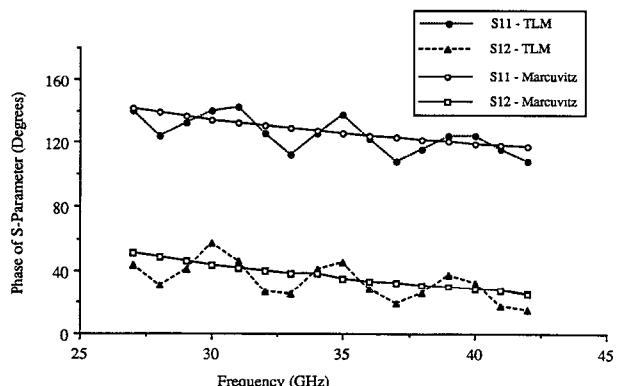
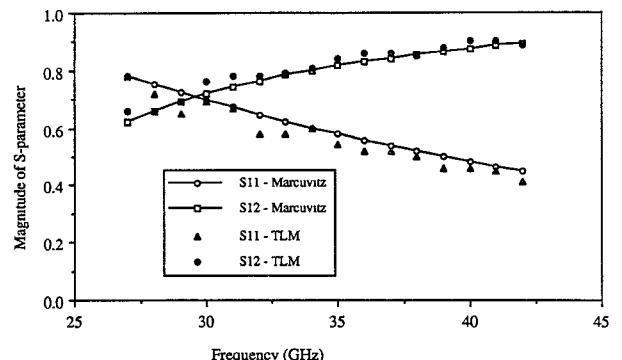


Fig. 4 : S-parameters of an Inductive Iris computed with regular Johns Matrix Absorbing boundaries

But to keep the reflections very small over a large bandwidth, the loss must increase slowly along the length of the waveguide. An alternative but equivalent solution is to increase  $\alpha$  with time. We found that by exponentially "tapering" the Johns Matrix ( $J(k)$ ) of the long uniform guide, this requirement can be met. The tapered Johns Matrix ( $J'(k)$ ) can be written as

$$J'(k) = J(k) e^{-\sum_{k'=1}^k \alpha(k')} \quad (5)$$

where  $\alpha(k')$  is

$$\alpha(k') = \alpha_1 e^{(\frac{(k'-1)}{N\Delta t} \ln(\alpha_2/\alpha_1))} \quad (6)$$

$\alpha_1$  is the attenuation constant for  $k=1$  (i.e. first iteration) and  $\alpha_2$  the attenuation constant for  $k=N\Delta t$ , the total number of terms in the Johns Matrix. We have optimized the values of  $\alpha_1$  and  $\alpha_2$  to get very small reflections over the operating bandwidth. The computed reflections of the two opposing absorbing boundaries separated by a WR28 waveguide section (about  $60 \Delta t$  long) are plotted in Fig. 3 for different combinations of  $\alpha_1$  and  $\alpha_2$ . It can be seen that in some cases, the reflections are less than one percent in the operating frequency band.

Using these absorbing boundary conditions, we have computed the S-parameters of a symmetrical inductive iris (of gap width equal to 3.46mm) in a WR28 waveguide. Results (shown in Fig. 5) compare well with those given in [6], and no ripple can be detected in both magnitude and phase. Also tapering leads to a considerable reduction in the size of the Johns Matrix (from 2000 values in the regular Johns Matrix to about less than 1000 values in the tapered Johns Matrix). Hence the time taken for the convolution using eqns. (1) and (2) is also reduced.

#### 4. RECURSIVE MODELING OF ABSORBING BOUNDARIES

The generation of Johns Matrix for absorbing waveguide boundaries has been discussed in section 2.1. This generation scheme requires a complete analysis of a big TLM mesh of length  $N\Delta t$ ; a very time-consuming process. We have found a much faster way to generate the Johns Matrix for an absorbing boundary by exploiting the fact that the Johns Matrix has a time dimension, i.e. is generated in discrete time step increments. In the following, we show how the new method works in the two-dimensional TLM case.

Let us assume for the moment that the Johns Matrix  $J[n]$  of a wideband waveguide matched load is already available. The impulse response of an  $n\Delta t$  long waveguide section, which is terminated with that Johns Matrix, would itself be the same Johns Matrix. (See Figure 6). This recursive property allows us to generate the unknown Johns Matrix in the following way.

Since the excitation signal reaches the end of the waveguide section with a delay of  $n\Delta t$ , the convolution process is shifted in time by that amount. Furthermore, the impulse response of the short waveguide section is identical to that of a long section for the first  $2n$  iterations. Hence, we can build the terminating Johns Matrix during the iteration process by simply transferring each impulse from the removed branch to the load side to be convolved with the incident impulses after a delay of  $n\Delta t$ . This is accomplished by assigning the impulse response  $R[n]$  to the Johns Matrix  $J[n]$  after each iteration, or simply by overlapping the memory area of the two matrices. This is a self-generating recursive algorithm. The above analysis also shows that the length of the TLM mesh for the Johns Matrix generation can be as short as  $1-\Delta t$ . The evolution of such a time-varying Johns Matrix is shown in Figure 7. A comparison of the Johns Matrices generated by the old and new methods is shown in Figure 8.

#### 5. PSEUDO-PARALLEL ITERATION ALGORITHM

The TLM algorithm is based on Huygens's principle and describes a parallel scattering phenomenon[2]. Because of the sequential computation in commonly available computers, the TLM algorithms were designed for iteration in a sequential manner. Such an algorithm is given in [2]. Not only fails this algorithm to take advantage of the parallel scattering property of the TLM method, but it also imposes a rigid order onto the co-ordinates of the computation domains, especially when the simulated structure involves inhomogeneous material. This is a serious drawback in designing a user-friendly program because the user does not want to keep track of the order of the computation domains.

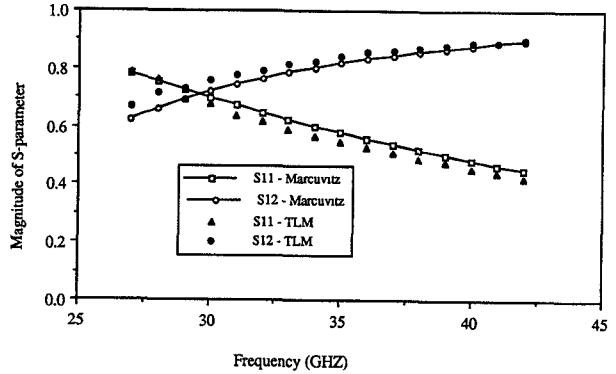


Fig. 5 : S-Parameters of an Inductive Iris computed with tapered Johns Matrix Absorbing Boundaries

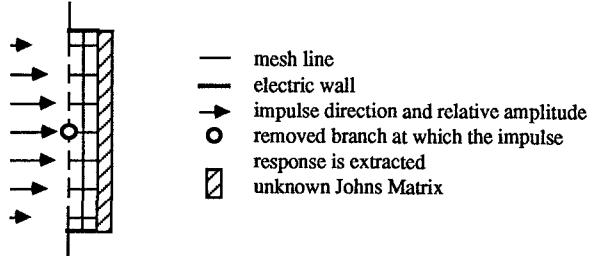


Fig. 6 : New Johns Matrix generation scheme. The length of the 2-D TLM mesh can be as short as  $1-\Delta t$ . An undefined Johns Matrix is used to terminate the mesh. The Johns Matrix is then built during the iteration process by simply transferring each impulse from the removed branch to the load side to be convolved with the incident impulses after a delay of  $1-\Delta t$ .

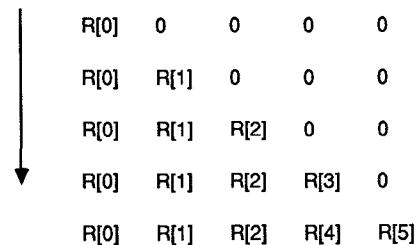


Fig. 7 : The evolution process of the time-varying Johns Matrix. The first 5 terms are shown.  $R[n]$  is the impulse response at  $t=n \Delta t$  which is equal to  $J[n]$ .

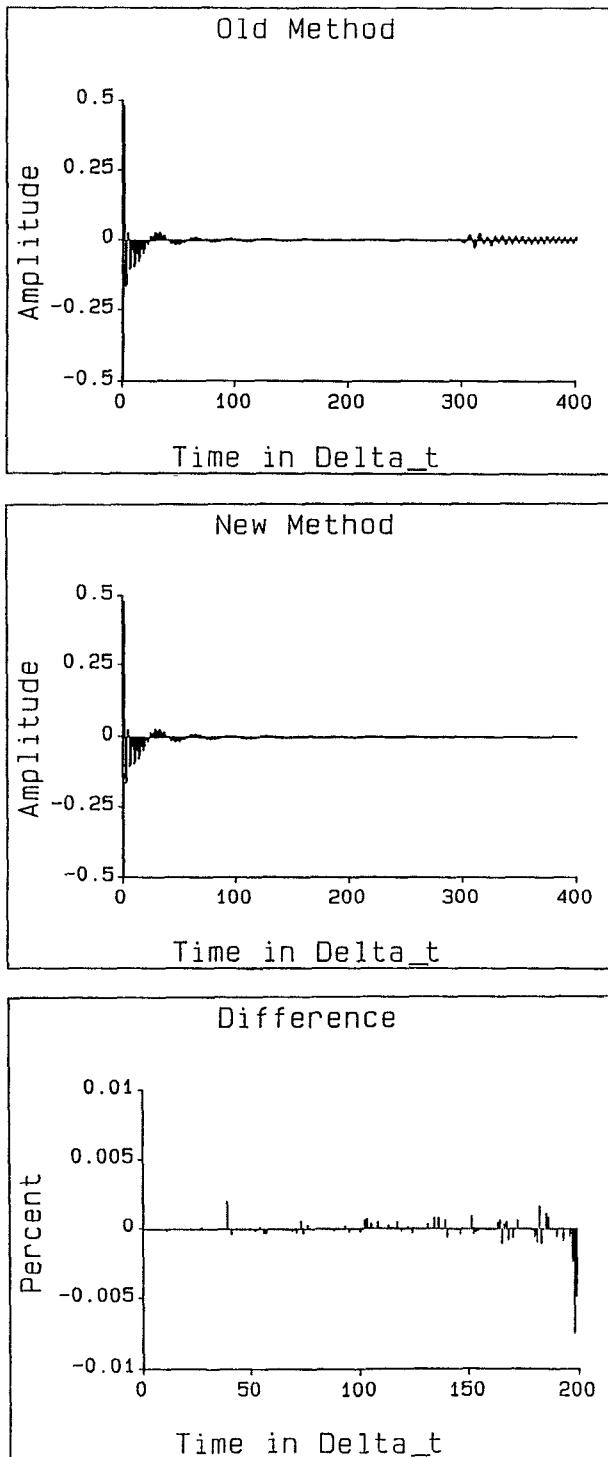


Fig. 8 : A comparison of the Johns Matrices generated by the old and new methods. The first Johns Matrix (top) is generated by using the old method. Because the structure used is only 100- $\lambda$  long, only the first 200 terms of the Johns Matrix are correct. The second Johns Matrix (middle) is generated by using the new recursive method. The graph at the bottom depicts the difference (in percent) between the two Johns Matrices.

To alleviate these problems, we have developed a pseudo-parallel iteration algorithm which allows us to input the computation domains in an arbitrary order. No sorting is required before performing the iteration process. This algorithm partitions the TLM mesh into a number of computation domains according to the input data, saves the impulse values entering each domain and then performs the traditional sequential iteration process within the domain. In a parallel computer, iteration can be performed in parallel over a number of computation domains. Therefore, as long as the size of the computation domains is approximately the same, and if the total number of computation domains is an integer multiple of the number of processors in the computer, this algorithm can fully exploit the power of a parallel computer.

## 6. CONCLUSION

We have developed and demonstrated a very efficient numerical model for wideband non-TEM absorbing boundaries for 3-D TLM, having less than one percent reflections over an entire waveguide operating band. It allows us to extract the scattering parameters of arbitrarily shaped three-dimensional discontinuities in waveguides from a single TLM simulation. The recursive procedure and pseudo-parallel iteration algorithm have also been in the latest version of our 2d-tlm simulator. The recursive Johs Matrix generation algorithm allows very easy and fast modeling of matching boundaries. The pseudo-parallel iteration algorithm allows the user interface to be made more user-friendly than before; the user can now input computation domains in any order and make changes at any time. All innovations presented in this paper represent considerable improvements over previous TLM procedures.

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